AN INVESTIGATION OF MASS TRANSFER IN A SPHERICAL CAPILLARY-POROUS BODY UNDER DRYING CONDITIONS

S. P. Rudobashta, A. N. Planovskii, and V. A. Svinarev

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This paper gives the results of an experimental investigation of moisture transport in a capillary-porous solid subject to convective drying.

The kinetics of drying capillary-porous bodies depends on the conditions of internal and external mass

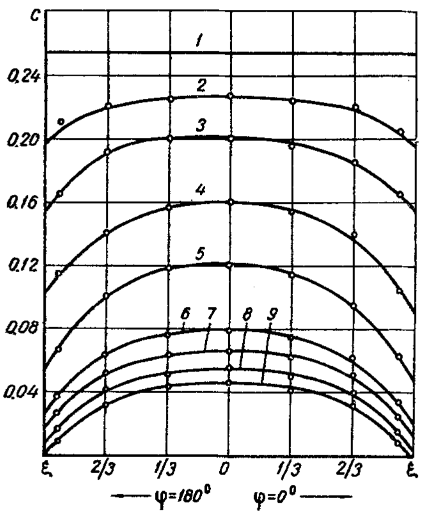

Fig. 1. Radial distribution of moisture in the sphere at different times: 1) τ = = 0 sec; 2) 1200; 3) 2400; 4) 4200; 5) 6000; 6) 8400; 7) 10800; 8) 13200; 9) 15600. C in kg/kg.

transfer; heat and mass transfer inside the body can be described by the following system of differential equations [1]:

$$c\gamma \frac{\partial t}{\partial \tau} = \nabla(\lambda \nabla t) + \varepsilon \nu \frac{\partial C}{\partial \tau}, \qquad (1)$$

$$\frac{\partial C}{\partial \tau} = \nabla(k \nabla C + k\delta \nabla t). \qquad (2)$$

In the general case, the coefficients k, δ, c, and ε are functions of the temperature and moisture content of the body and, hence, an exact solution of the system of equations (1)–(2) can be obtained only by using specific relationships $k = f_1(C, t)$, $\delta = f_2(C, t)$, $\varepsilon = f_3(C, t)$, and $c = f_4(C, t)$.

The separate effect of temperature and moisture content on the coefficient k was investigated in [2, 3], and it was found that the mass transfer decreased with reduction of moisture content and increased with increase in temperature. In actual drying the temperature of a body usually increases with reduction in moisture content, and, hence, it is of interest to investigate the change in mass transfer under the influence of these opposing parameters. If the ultimate result is an insignificant increase in k, then k can be regarded as constant in first approximation and this justifies the calculation of drying processes from analytical relationships obtained for the case of constant coefficients [1]. In the opposite case these relationships will be applicable only in small ranges of moisture content.

In the present work, we investigated the mass transfer in a spherical capillary-porous body subjected to drying in a forced flow of air. The experiments were conducted in a closed-circuit air tunnel 300 mm in diameter with an airflow velocity of 4.7 m/sec, a relative humidity φ = 6%, and a temperature of 323° K. The experimental body [a plaster (gypsum) sphere with a density of 1500 kg/m³] consisted of three parts: two segments and a disk enclosed between them. The disk and one segment together formed one hemisphere; the contact surfaces were carefully finished. A 2-mm-thick layer of soft rubber was cemented to the flat surface of the second segment. The disk was 4 mm thick. All the parts of the sphere were held together by a clamp, and the layer of soft rubber ensured uniform pressure on the disk.

The sphere was impregnated with water in a vacuum and was placed in the dryer so that the joint planes were parallel to the axis of the airflow. The axial symmetry of the "working" hemisphere (disk-segment) permitted the moisture distribution to behave in the same way as it would if it had been part of an integral sphere without a rubber partition.

After the elapse of a certain time the sphere was removed from the dryer; several fragments were chipped from the disk and immediately placed in weighing bottles with ground stoppers. The time from removal of the sphere from the dryer to the time of closing the last bottle was not more than 20 sec. The moisture content of the samples was determined on an ADV-200 analytical balance. Since the plaster was not distinguished by high resistance to water (in one experiment the specimen lost 0.3% of its weight), we replaced the "working" segment with a new one after every two experiments.

From the obtained data we plotted the moisture distribution in the sphere at different times (Fig. 1). There was some spread of the calculated values of local moisture content. (This can evidently be attributed to the fact that, although they had the same volume of open pores, the individual disks and the segment could still differ slightly from one another in the radial distribution of the pores.) Despite this, averaging of the data for six experiments gave, as Fig. 1 shows, a rather distinct picture of the radial distribution of

Local Values of Temperature and Moisture Content in Sphere

τ, sec	ξ	Moisture content C, kg/kg for φ (deg) equal to		$C_R, \varphi=108° - C_R, \varphi=0°$	Temperature, °K for φ (deg) equal to				
		721	108		0	45	90	135	180
1200	0	0.225	0.225	0	23.2	23.2	23.2	23.2	23.2
	1/3	0.222	0.225		23.2	23.2	23.2	23.2	23.2
	2/3	0.220	0.220		23.2	23.2	23.2	23.2	23.2
	11/12	0.205	0.205		23.4	23.2	23.2	23.2	23.2
2400	0	0.200	0.200	0.013	24.5	24.5	24.5	24.5	24.5
	1/3	0.200	0.200		24.7	24.7	24.5	24.5	24.5
	2/3	0.189	0.195		25.0	24.9	24.4	24.4	24.4
	11/12	0.164	0.175		25.4	25.4	24.1	24.2	24.3
4200	0	0.160	0.160	0.036	26.6	26.6	26.6	26.6	26.6
	1/3	0.156	0.159		27.1	26.9	26.5	26.5	26.1
	2/3	0.141	0.148		28.4	27.8	26.1	26.0	26.1
	11/12	0.117	0.132		29.6	28.5	25.5	25.4	26.3
6000	0	0.120	0.120	0.225	31.3	31.3	31.3	31.3	31.3
	1/3	0.117	0.119		32.7	32.6	31.5	30.6	30.6
	2/3	0.105	0.105		33.3	33.7	31.0	29.6	30.7
	11/12	0.081	0.089		34.5	34.9	30.3	28.7	31.5
8400	0	0.079	0.079	0.003	41.4	41.4	41.4	41.4	41.4
	1/3	0.076	0.076		42.0	41.7	41.4	41.2	41.2
	2/3	0.062	0.065		42.6	42.2	41.1	40.6	41.3
	11/12	0.033	0.038		43.0	42.5	40.7	40.1	41.6
10800	0	0.066	0.066	0.001	44.8	44.8	44.8	44.8	44.8
	1/3	0.063	0.062		45.0	45.1	44.8	44.7	44.6
	2/3	0.051	0.051		45.2	45.4	44.7	44.4	44.6
	11/12	0.024	0.025		45.4	45.4	44.4	44.1	44.7
220	0	0.055	0.055	0	46.1	46.1	46.1	46.1	46.1
	1/3	0.053	0.056		46.4	46.4	46.1	46.1	46.1
	2/3	0.041	0.042		46.4	46.5	46.1	46.1	46.1
	11/12	0.015	0.016		46.6	46.6	45.7	45.7	46.1

Note. In view of the compactness of the material the table gives the temperatures for ξ = 11/12 instead of the experimentally measured temperatures on the surface of the sphere.

moisture content in the sphere. The distribution curves were recorded for the following angles of attack of the airflow: $\varphi = 0$, 72, 108, and 180°. (By angle of attack we mean the angle between the direction of the airflow and the normal to the sphere surface at the given point.) An analysis of the obtained data showed their complete agreement at the control point (moisture content in center of sphere). Each curve can be fairly accurately represented by the equation of a quadratic parabola. The maximum point of the curves, whose position is determined by the ratio of the external diffusion resistances of the front and rear parts of the sphere (in our case it was 1.22), was shifted slightly to the rear.

An examination of Table 1 and Fig. 1 together indicates the degree of asymmetry of the problem at different times. The asymmetry developed gradually and reached a maximum at $\tau = 4800$ sec ($\bar{C} = 0.114$). The greatest surface moisture content was observed at $\varphi = 108°$, i.e., at points where the mass transfer coefficients were lowest. Beginning at $\tau = 8400$ sec ($\bar{C} = 0.052$) the problem can be regarded as perfectly symmetrical; this is due to the fact that at $\bar{C} < 0.052$, the process is controlled completely by internal diffusion (this last conclusion was derived from an analysis of the drying-rate curves recorded for different velocities of airflow: at $\bar{C} = 0.05$ all the curves merged into one).

In addition to the moisture distributions, we also recorded the temperature distributions in the sphere. For this purpose we cast two plaster hemispheres. Along the diameter of one of them we placed six thermocouples, from which two leads were brought out to the surface. The hemispheres were then joined together by a plaster solution made up with the same proportion of water as the hemispheres (40 parts of water to 100 parts of plaster by weight). The sphere, mounted on an ebonite holder, was impregnated with water and installed in the dryer; the thermocouple leads emerging from the sphere at one point passed through a hole in the column and were connected to a potentiometer. Thus, temperature measurement began 300 sec after the start of drying. We carried out three experiments, each with three different spheres for each angle of attack; the values of the angles of attack in the experiments were 0, 45, 90, 135, and 180°.

Like the moisture curves, the temperature curves for different φ agreed perfectly at the control point (center of sphere). An examination of the temperature curves showed that the temperature distribution in a moist sphere drying in a forced air flow was much more complex than the moisture distribution: for angles of attack of 0, 45, and 180°, the surface temperature gradients were directed toward the outside of the sphere, while for angles of attack of 90 and 135°, they were directed inward; for angles of attack of 0, 45, 90, and 135°, they retained the same sign when the considered point moved along the radius, and for $\varphi = 180°$ the sign changed when the value of ξ was approximately 0.5. At individual points, the temperature gradients reached a high value (to 180–200° K/m) and, hence, could have a great effect on the movement of moisture inside the material.

Comparing the distribution of the temperature gradients and local mass transfer coefficients relative to the coordinate φ, we can come to this conclusion: the position of the temperature gradients is such that they enhance the nonuniformity of the moisture distribution due to the variation of the mass transfer coefficient with φ.

At points where the coefficient β had high values ($\varphi = 0$–45, 180°), the temperature gradients created an additional resistance to the moisture flux, thus leading to more rapid drying of the surface layers, whereas at points with a low value of β ($\varphi = 90$–135°), the temperature gradients created an additional driving force for transport of moisture to the surface.

Before proceeding to the quantitative analysis of the obtained data, let us mention that we recorded several thermograms for a sphere with a 2-mm-thick rubber interlayer over the plane with the thermocouples (to estimate the heat flux through the rubber partition in the moisture-distribution experiments). We did not notice any significant distortion of the temperature distribution. (The thermocouples situated at a distance of 2 mm from the sphere surface showed an increase of 0.3° K during the first 240 sec.)

We write the basic mass transfer equation for an element dF of the sphere surface:

$$i = -k_R \gamma \left[\left(\frac{\partial C}{\partial r} \right)_R + \delta_R \left(\frac{\partial t}{\partial r} \right)_R \right]. \quad (3)$$

We integrate both parts of the equation from 0 to F and divide by F and, then, after introducing the quantities $k_{R,av}$ and $\delta_{R,av}$ (the average of coefficients k and δ_R over the surface F), we have

$$\bar{i} = -k_{R,\,av} \gamma \left[\frac{1}{F} \int_0^F \left(\frac{\partial C}{\partial r} \right)_R dF + \delta_{R,\,av} \frac{1}{F} \int_0^F \left(\frac{\partial t}{\partial r} \right)_R dF \right] = -k_{R,\,av} \gamma \left[\left(\frac{\partial C}{\partial r} \right)_{R,\,av} + \delta_{R,\,av} \left(\frac{\partial t}{\partial r} \right)_{R,\,av} \right]. \quad (4)$$

The values of $(\partial C/\partial r)_R$ and $(\partial t/\partial r)_R$ were found by graphic differentiation of the curves $C = f_1(r, \tau)$, $t = f_2(r, \tau)$ for $r = R$. To correlate the values of the local gradients, we plotted the graphs of (Fig. 2)

$$\left(\frac{\partial C}{\partial r} \right)_{R,\,\varphi} = F_1(\tau) \text{ and } \left(\frac{\partial t}{\partial r} \right)_{R,\,\varphi} = F_2(\tau).$$

We found $(\partial C/\partial r)_{R,av}$ and $(\partial t/\partial r)_{R,av}$ graphically, as shown in Fig. 3; corrected values of the local gradients were plotted on the y-axis.

It should be noted that despite the fairly high values of $(\partial t/\partial r)_R$ (up to 180–200° K/m), their mean integral values did not exceed 18° K/m. If we assume, on the basis of the data of [4], that the coefficient δ lies in the range 0–0.01° K/m, we find that $\delta_{max}(\partial t/\partial r)_{R,av.\,max} = 0.18$ 1/m, and the ratio $\delta_{max}(\partial t/\partial r)_{R,av.max}/(\partial C/\partial r)_{R,av}$ lies in the range 0.03–0.06. It is very unlikely

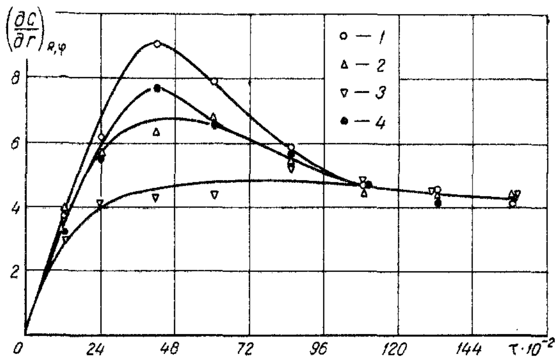

Fig. 2. Correlation graph for moisture gradients: 1) $\varphi = 0°$; 2) 72°; 3) 108°; 4) 180°. τ in sec.

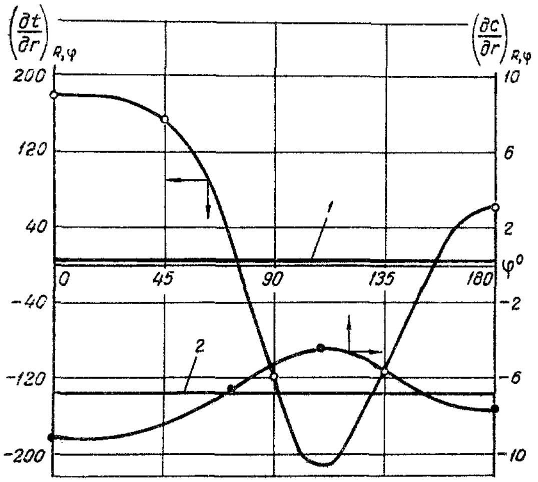

Fig. 3. Example of determination of mean integral values of temperature and moisture gradients at $\tau = 4200$ sec: 1) $(\partial t/\partial r)_{R,av}$, °K/m; 2) $(\partial C/\partial r)_{R,av}$, 1/m.

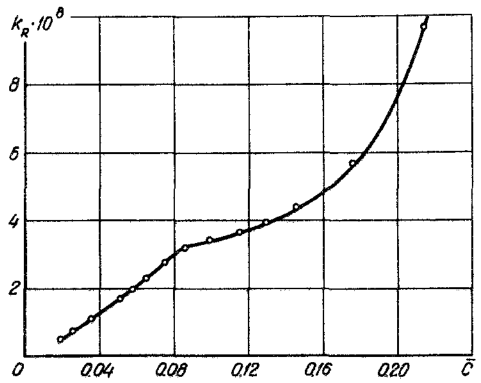

Fig. 4. Variation of $k_{R,av}$ of plaster sphere drying at air temperature 323° K. C in kg/kg, k_R in m^2/sec.

that δ_{max} corresponds to $(\partial t/\partial r)_{R,av.max}$, and, hence, the contribution of the second term in Eq. (4) is very small.

From the considered example, we can draw the following general conclusion: in cases in which the boundary conditions of mass transfer are nonuniform, there may be a distribution of temperature gradients over the surface of the body such that, despite their considerable local values, the total thermal moisture conduction is small. This may occur in the case of drying of single bodies, since for a body in a bed, as investigations showed, the boundary conditions can be regarded as uniform.

We used Eq. (4) to calculate $k_{R,av}$, which in this case is a regime parameter. The variation of $k_{R,av}$ under the action of two parameters (temperature and moisture content) in relation to the average moisture content of the material is shown in Fig. 4. As the figure shows, when C changes from the initial value $C_i = 0.255$ kg/kg to $\bar{C} = 0.02$ kg/kg, the mass transfer $k_{R,av}$ changes by a factor of 20, and, hence, solution of the system of equations (1)–(2) by assuming k = const would not correspond to actual conditions in this case.

Yet replacement of the asymmetric problem by a symmetric one by the introduction of average values of $k_{r,av}$ and $\delta_{r,av}$ over the spherical surfaces simplifies the initial equations (1) and (2), first, by allowing consideration of the one-dimensional problem instead of the two-dimensional one and, second, by the fact that, in this one-dimensional problem, thermal moisture conduction can be neglected. We must stipulate, however, that such an operation is valid when it is a problem of finding a relationship $\bar{C} = f(\tau)$. In addition, the question of how general the condition $\delta_{r,av}(\partial t/\partial r)_{r,av} \approx 0$ is for the asymmetric problem requires experimental verification.

The obtained curves for the moisture distribution in the sphere were used for an indirect verification of the correctness of determining local moisture contents. Since samples for moisture content were always taken from the disk, in the case of bad contact between the disk and segment the mean integral moisture content calculated from the experimental curves would differ from the values taken from the drying curve of an integral sphere. We found, however, that the weight of moisture in the sphere at any time, determined by direct weighing, was exactly the same as the weight found by graphic integration of the moisture distribution curves for the same times.

NOTATION

β-is the mass transfer coefficient; C is the local moisture content of material, kg/kg; $\bar{C}$ is the volume average moisture content, kg/kg; δ is the coefficient of thermal moisture conduction, °K/m; ε is the phase change criterion; c is the specific heat; γ is the density of the material, kg/m^3; F is the surface of the sphere; i is the moisture flux density, kg/m^2 · sec; $\bar{i}$ is the average moisture flux density over the surface of the sphere; k is the mass transfer, m^2/sec; $k_{R,av}$ is the average effective mass transfer over the surface of the sphere, m^2/sec; λ is the thermal conductivity; r is the instantaneous radius; R is the radius of the sphere; t is the local temperature, °K; τ is the time; ν is the heat of vaporization; ξ is the ratio of the instantaneous radius to the sphere radius; φ is the angle of attack of the airflow. Subscripts: i is the initial value; av is the average value; R refers to the surface of the sphere.

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Institute of Chemical Engineering, Moscow